

Cole-Hopf 変換を用いたバーガス流体統計量の量子計算

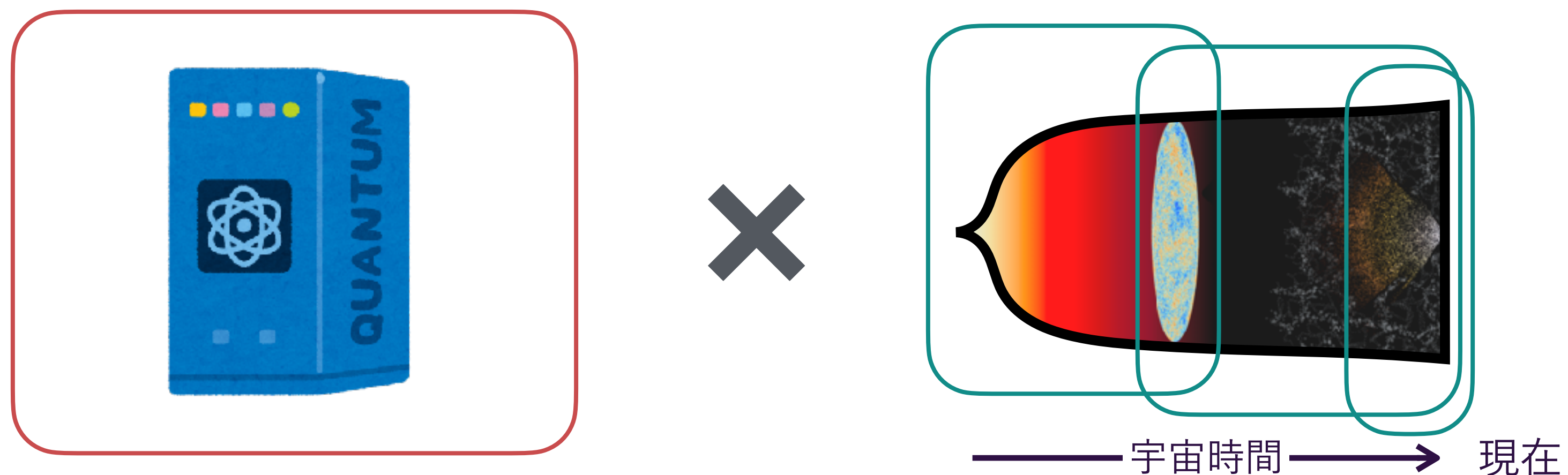
Based on [\[arXiv:2412.17206\]](#) in collaboration with

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2026/5/8, 第3回 Quantum CAE 研究会
Fumio Uchida (CTPU-CGA, IBS)



量子計算を宇宙物理に応用したい



自己紹介

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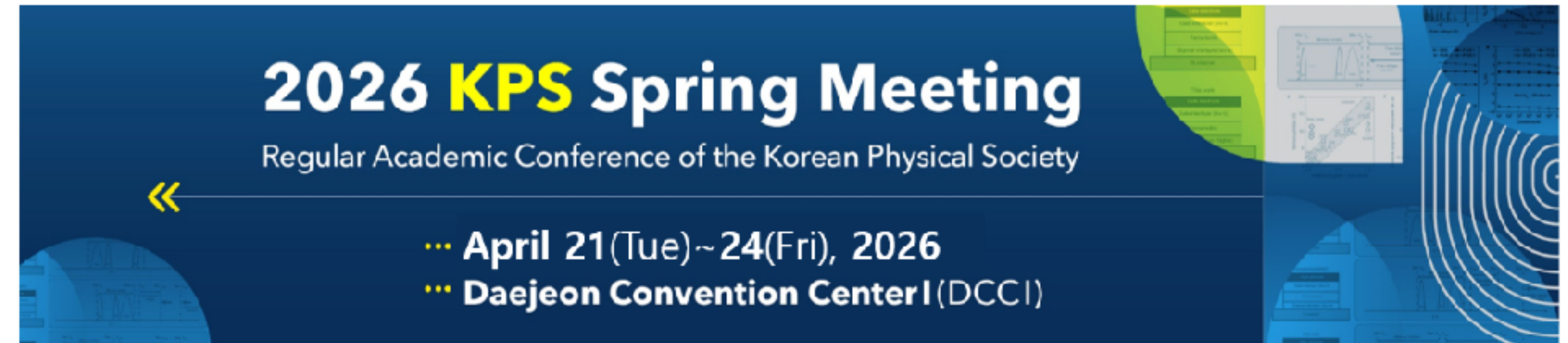
藤澤 幸太郎 工学院大学

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韓国物理学会 (余談)

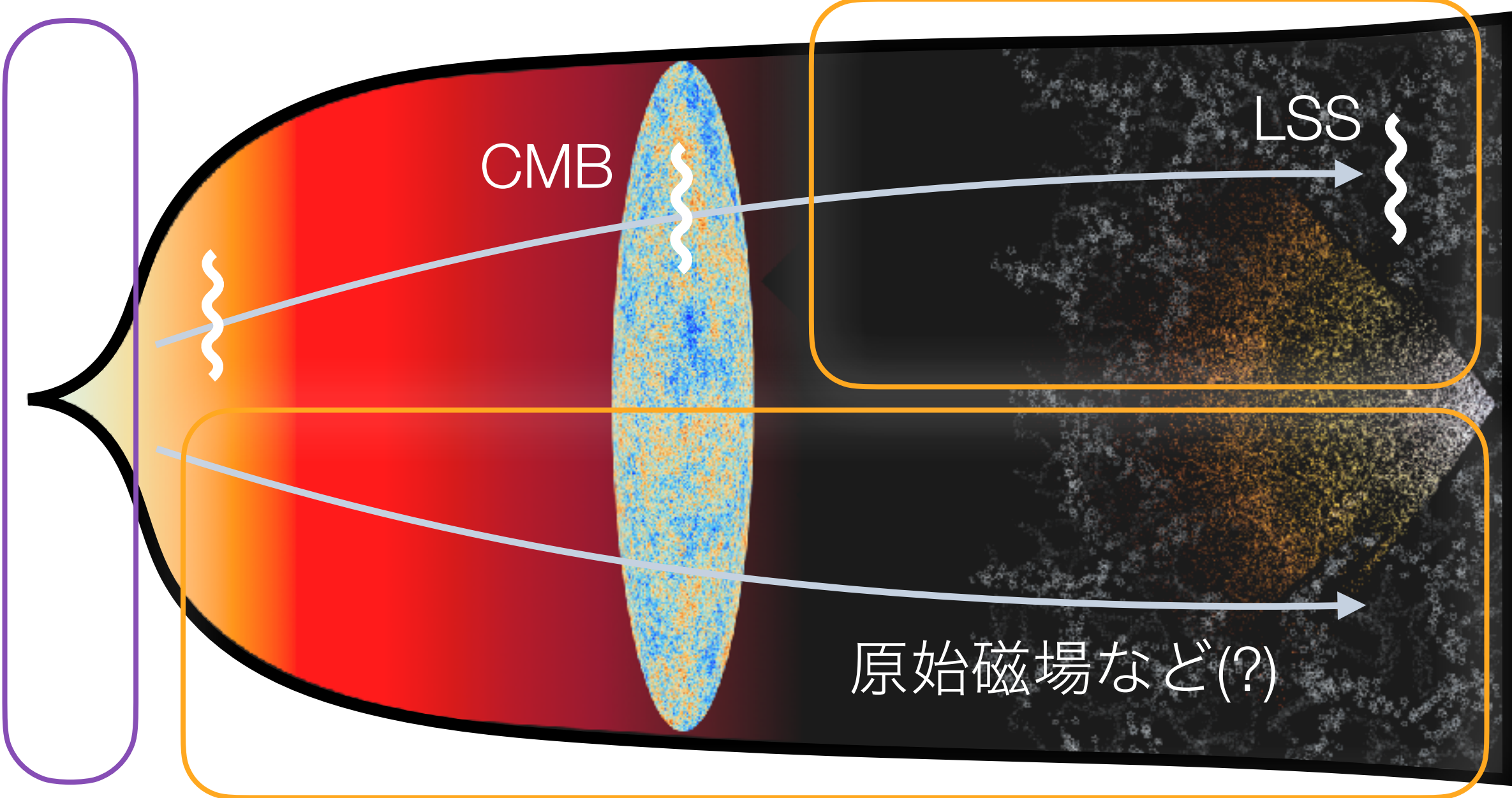
全日程に量子計算セッション

- 中性原子型 [Focus session]
- 超伝導型
- イオントラップ型
- フォトン型
- 量子情報
- 量子アルゴリズム

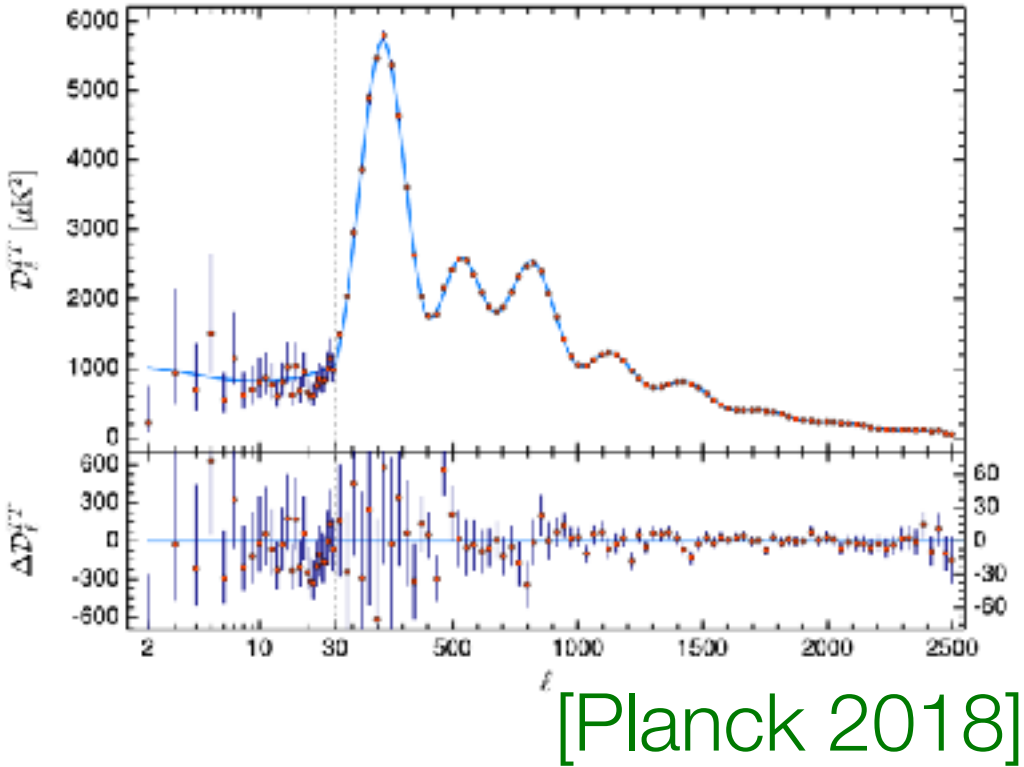


Session	Room	101	102	103	104	105	106	107	108	201	202	204	206	208	301	
04.21 (Tue)	Session BT	BT1-01 [I] Status of Neutrons	BT2-01 [I] Quantum Entanglement in Quantum Optics	BT3-01 [I] Quantum Entanglement in Quantum Optics	BT4-01 [I] Quantum Entanglement in Quantum Optics	BT5-01 [I] Quantum Entanglement in Quantum Optics	BT6-01 [I] Quantum Entanglement in Quantum Optics	BT7-01 [I] Quantum Entanglement in Quantum Optics	BT8-01 [I] Quantum Entanglement in Quantum Optics	BT9-01 [I] Quantum Entanglement in Quantum Optics	BT10-01 [I] Quantum Entanglement in Quantum Optics	BT11-01 [I] Quantum Entanglement in Quantum Optics	BT12-01 [I] Quantum Entanglement in Quantum Optics	BT13-01 [I] Quantum Entanglement in Quantum Optics		
04.22 (Wed)	Session B	B1-01 [I] Superconducting Environment	B2-01 [I] Quantum Optics	B3-01 [I] Neutron I	B4-01 [I] QFT for Experimental Research I	B5-01 [I] Nano and Mesoscopic Physics	B6-01 [I] Energy 20 I	B7-01 [I] Semiconductors I	B8-01 [I] Photonics & Optics I	B9-01 [I] Single Photon	B10-01 [I] Non-accelerator I	B11-01 [I] FEA, ML, Accel., Fusion, Pos. I	B12-01 [I] Current Applied Physics Emerging Topics I	B13-01 [I] FEA, Coll. Astrophysics	B14-01 [I] CW & PWS I	B15-01 [I] Accelerator I
	Session C	C1-01 [I] Quantum Optics	C2-01 [I] Quantum Optics	C3-01 [I] Neutron II	C4-01 [I] QFT for Experimental Research II	C5-01 [I] Nano and Mesoscopic Physics	C6-01 [I] Energy 20 II	C7-01 [I] Semiconductors II	C8-01 [I] Photonics & Optics II	C9-01 [I] Single Photon	C10-01 [I] Non-accelerator II	C11-01 [I] FEA, ML, Accel., Fusion, Pos. II	C12-01 [I] Current Applied Physics Emerging Topics II	C13-01 [I] FEA, Coll. Astrophysics	C14-01 [I] CW & PWS II	C15-01 [I] Accelerator II
	Session D	D1-01 [I] Quantum Optics	D2-01 [I] Quantum Optics	D3-01 [I] Neutron III	D4-01 [I] QFT for Experimental Research III	D5-01 [I] Nano and Mesoscopic Physics	D6-01 [I] Energy 20 III	D7-01 [I] Semiconductors III	D8-01 [I] Photonics & Optics III	D9-01 [I] Single Photon	D10-01 [I] Non-accelerator III	D11-01 [I] FEA, ML, Accel., Fusion, Pos. III	D12-01 [I] Current Applied Physics Emerging Topics III	D13-01 [I] FEA, Coll. Astrophysics	D14-01 [I] CW & PWS III	D15-01 [I] Accelerator III
	Session ET	ET1-01 [I] Education Committee Lecture	ET2-01 [I] Education Committee Lecture	ET3-01 [I] Neutron III	ET4-01 [I] QFT for Experimental Research III	ET5-01 [I] Nano and Mesoscopic Physics	ET6-01 [I] Energy 20 III	ET7-01 [I] Semiconductors III	ET8-01 [I] Photonics & Optics III	ET9-01 [I] Single Photon	ET10-01 [I] Non-accelerator III	ET11-01 [I] FEA, ML, Accel., Fusion, Pos. III	ET12-01 [I] Current Applied Physics Emerging Topics III	ET13-01 [I] FEA, Coll. Astrophysics	ET14-01 [I] CW & PWS III	ET15-01 [I] Accelerator III
04.23 (Thu)	Session F	F1-01 [I] Neutron and surface	F2-01 [I] Quantum Optics	F3-01 [I] Neutron I	F4-01 [I] QFT for Experimental Research I	F5-01 [I] Nano and Mesoscopic Physics	F6-01 [I] Energy 20 I	F7-01 [I] Semiconductors I	F8-01 [I] Photonics & Optics I	F9-01 [I] Single Photon	F10-01 [I] Non-accelerator I	F11-01 [I] FEA, ML, Accel., Fusion, Pos. I	F12-01 [I] Current Applied Physics Emerging Topics I	F13-01 [I] FEA, Coll. Astrophysics	F14-01 [I] CW & PWS I	F15-01 [I] Accelerator I
	Session Y									Y1-01 [I] Honorary Lecture						
	Session D	D1-01 [I] Quantum Optics	D2-01 [I] Quantum Optics	D3-01 [I] Neutron II	D4-01 [I] QFT for Experimental Research II	D5-01 [I] Nano and Mesoscopic Physics	D6-01 [I] Energy 20 II	D7-01 [I] Semiconductors II	D8-01 [I] Photonics & Optics II	D9-01 [I] Single Photon	D10-01 [I] Non-accelerator II	D11-01 [I] FEA, ML, Accel., Fusion, Pos. II	D12-01 [I] Current Applied Physics Emerging Topics II	D13-01 [I] FEA, Coll. Astrophysics	D14-01 [I] CW & PWS II	D15-01 [I] Accelerator II
	Session HT	HT1-01 [I] Honorary Lecture	HT2-01 [I] Honorary Lecture	HT3-01 [I] Neutron III	HT4-01 [I] QFT for Experimental Research III	HT5-01 [I] Nano and Mesoscopic Physics	HT6-01 [I] Energy 20 III	HT7-01 [I] Semiconductors III	HT8-01 [I] Photonics & Optics III	HT9-01 [I] Single Photon	HT10-01 [I] Non-accelerator III	HT11-01 [I] FEA, ML, Accel., Fusion, Pos. III	HT12-01 [I] Current Applied Physics Emerging Topics III	HT13-01 [I] FEA, Coll. Astrophysics	HT14-01 [I] CW & PWS III	HT15-01 [I] Accelerator III
	Session I	I1-01 [I] Quantum Optics	I2-01 [I] Quantum Optics	I3-01 [I] Neutron III	I4-01 [I] QFT for Experimental Research III	I5-01 [I] Nano and Mesoscopic Physics	I6-01 [I] Energy 20 III	I7-01 [I] Semiconductors III	I8-01 [I] Photonics & Optics III	I9-01 [I] Single Photon	I10-01 [I] Non-accelerator III	I11-01 [I] FEA, ML, Accel., Fusion, Pos. III	I12-01 [I] Current Applied Physics Emerging Topics III	I13-01 [I] FEA, Coll. Astrophysics	I14-01 [I] CW & PWS III	I15-01 [I] Accelerator III
04.24 (Fri)	Session J	J1-01 [I] Quantum Optics	J2-01 [I] Quantum Optics	J3-01 [I] Neutron III	J4-01 [I] QFT for Experimental Research III	J5-01 [I] Nano and Mesoscopic Physics	J6-01 [I] Energy 20 III	J7-01 [I] Semiconductors III	J8-01 [I] Photonics & Optics III	J9-01 [I] Single Photon	J10-01 [I] Non-accelerator III	J11-01 [I] FEA, ML, Accel., Fusion, Pos. III	J12-01 [I] Current Applied Physics Emerging Topics III	J13-01 [I] FEA, Coll. Astrophysics	J14-01 [I] CW & PWS III	J15-01 [I] Accelerator III
	Session K	K1-01 [I] Quantum Optics	K2-01 [I] Quantum Optics	K3-01 [I] Neutron III	K4-01 [I] QFT for Experimental Research III	K5-01 [I] Nano and Mesoscopic Physics	K6-01 [I] Energy 20 III	K7-01 [I] Semiconductors III	K8-01 [I] Photonics & Optics III	K9-01 [I] Single Photon	K10-01 [I] Non-accelerator III	K11-01 [I] FEA, ML, Accel., Fusion, Pos. III	K12-01 [I] Current Applied Physics Emerging Topics III	K13-01 [I] FEA, Coll. Astrophysics	K14-01 [I] CW & PWS III	K15-01 [I] Accelerator III
	Session L	L1-01 [I] Quantum Optics	L2-01 [I] Quantum Optics	L3-01 [I] Neutron III	L4-01 [I] QFT for Experimental Research III	L5-01 [I] Nano and Mesoscopic Physics	L6-01 [I] Energy 20 III	L7-01 [I] Semiconductors III	L8-01 [I] Photonics & Optics III	L9-01 [I] Single Photon	L10-01 [I] Non-accelerator III	L11-01 [I] FEA, ML, Accel., Fusion, Pos. III	L12-01 [I] Current Applied Physics Emerging Topics III	L13-01 [I] FEA, Coll. Astrophysics	L14-01 [I] CW & PWS III	L15-01 [I] Accelerator III
	Session M	M1-01 [I] Quantum Optics	M2-01 [I] Quantum Optics	M3-01 [I] Neutron III	M4-01 [I] QFT for Experimental Research III	M5-01 [I] Nano and Mesoscopic Physics	M6-01 [I] Energy 20 III	M7-01 [I] Semiconductors III	M8-01 [I] Photonics & Optics III	M9-01 [I] Single Photon	M10-01 [I] Non-accelerator III	M11-01 [I] FEA, ML, Accel., Fusion, Pos. III	M12-01 [I] Current Applied Physics Emerging Topics III	M13-01 [I] FEA, Coll. Astrophysics	M14-01 [I] CW & PWS III	M15-01 [I] Accelerator III

宇宙論と統計量



CMB 温度ゆらぎのパワースペクトル



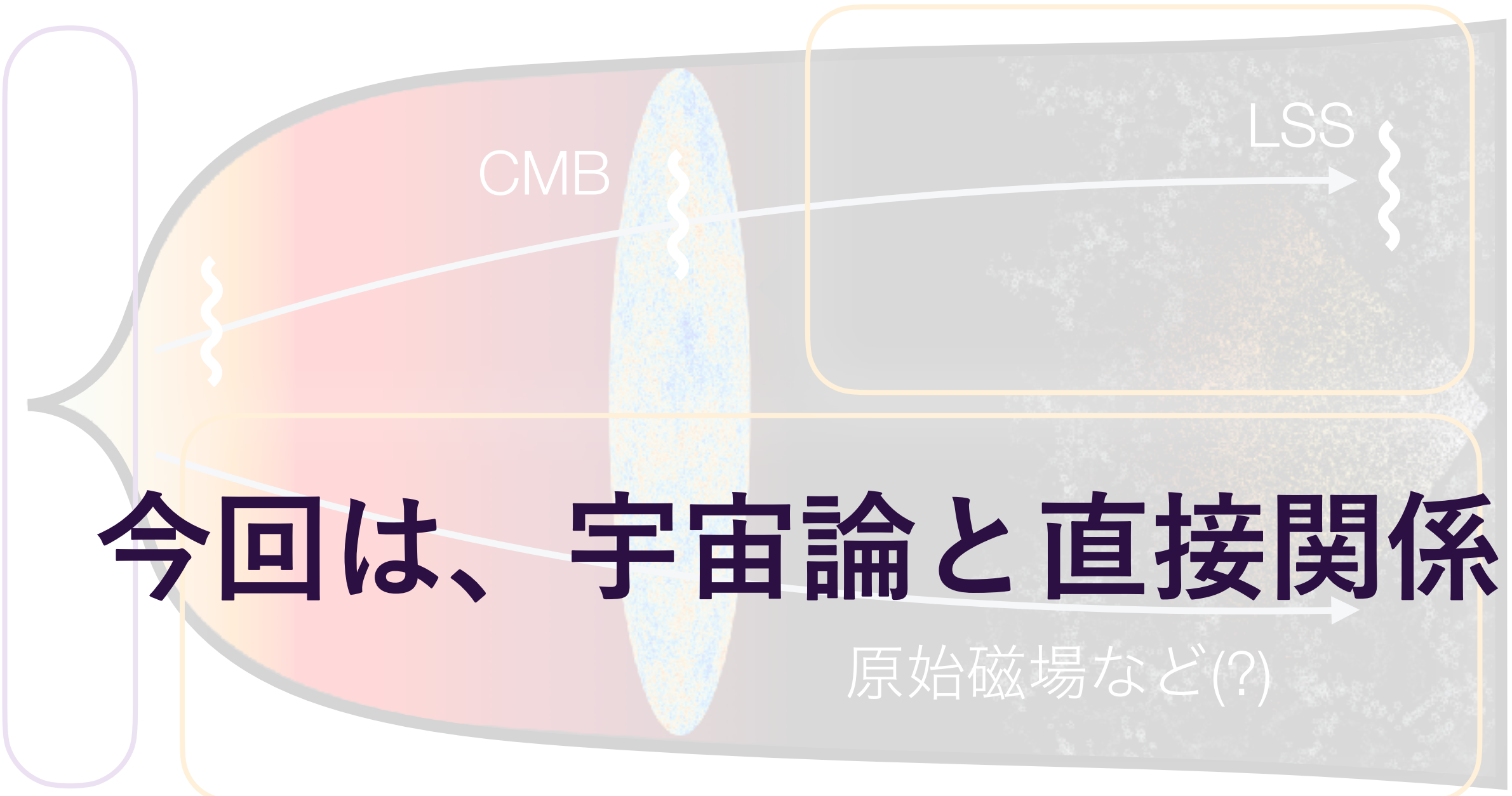
理論モデルは古典ゆらぎの統計的性質を決定

非線形時間発展 (密度ゆらぎが重力でつぶれる など)

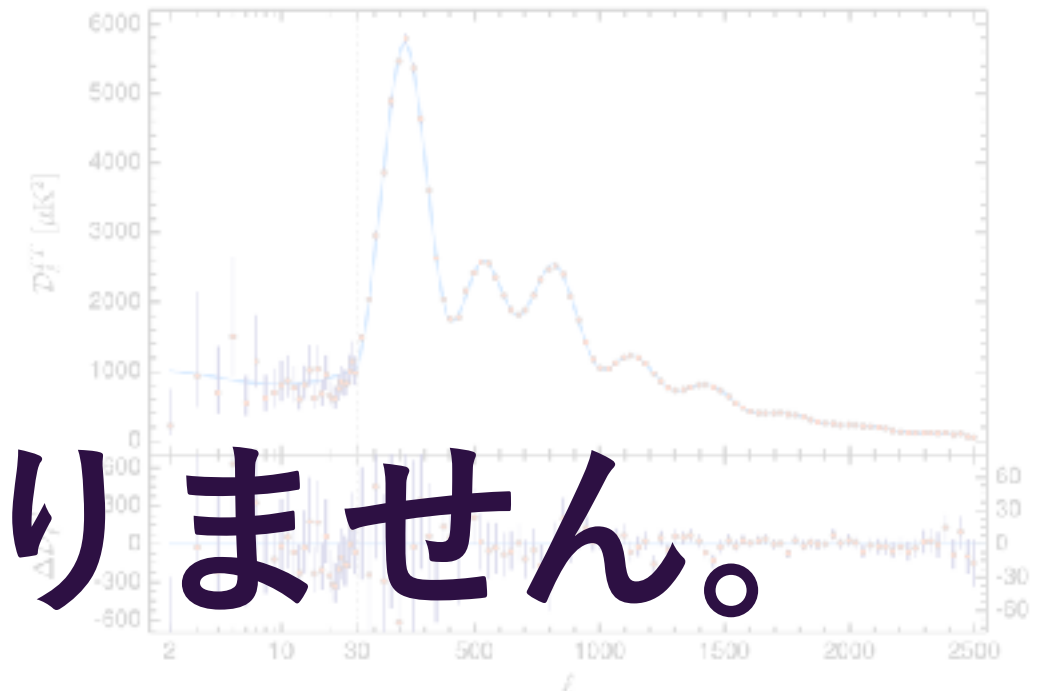
観測と理論の整合性を検討

Figs (modified) from [Planck, ESA], [D. Schlegel/Berkeley Lab using data from DESI, M. Zamani (NSF's NOIRLab)]

宇宙論と統計量



CMB 温度ゆらぎのパワースペクトル



今回は、宇宙論と直接関係ありません。

理論モデルは古典ゆらぎの統計的性質を決定

非線形時間発展 (密度ゆらぎが重力でつぶれる など)

観測と理論の整合性を検討

Figs (modified) from [Planck, ESA], [D. Schlegel/Berkeley Lab using data from DESI, M. Zamani (NSF's NOIRLab)]

本研究の位置づけ

宇宙論でしばしば現れる問題

- **非線形偏微分方程式**を解き、
- 解の**統計的性質**を調べたい。

今回は、1次元 Burgers 方程式

$$\partial_t u + u \partial_x u = \nu \partial_x^2 u$$

移流 拡散

のもとで、速度場の n 次モーメント

$$\langle u^n \rangle$$

を**量子計算**できることを示す。

流体の toy model としての Burgers 方程式

Burgers equation

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \nu \nabla^2 \mathbf{u} \quad [\text{Bateman 1915}] [\text{Burgers 1948}]$$

移流 拡散

+ pressure gradient

Navier–Stokes equation

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{u}$$

+ electromagnetic field

magneto-hydrodynamics (MHD)

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{\rho} - \frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{u}$$

1次元 Burgers 方程式は拡散方程式に帰着する

1次元 Burgers 方程式

$$\partial_t u + u \partial_x u = \nu \partial_x^2 u$$

1次元拡散方程式

$$\partial_t \psi = \nu \partial_x^2 \psi$$



Cole-Hopf 変換 [Hopf 1950] [Cole 1951]

$$\psi = \exp\left(-\frac{1}{2\nu} \int^x dy u(y)\right), \quad u = -2\nu \frac{\partial_x \psi}{\psi}$$

非線形変数変換 × 量子計算

1次元Burgers方程式：非線形偏微分方程式

Cole-Hopf 変換：非線形変数変換

1次元拡散方程式：線形偏微分方程式

空間離散化のち量子計算で時間積分

量子状態から、解の統計量を読み出し

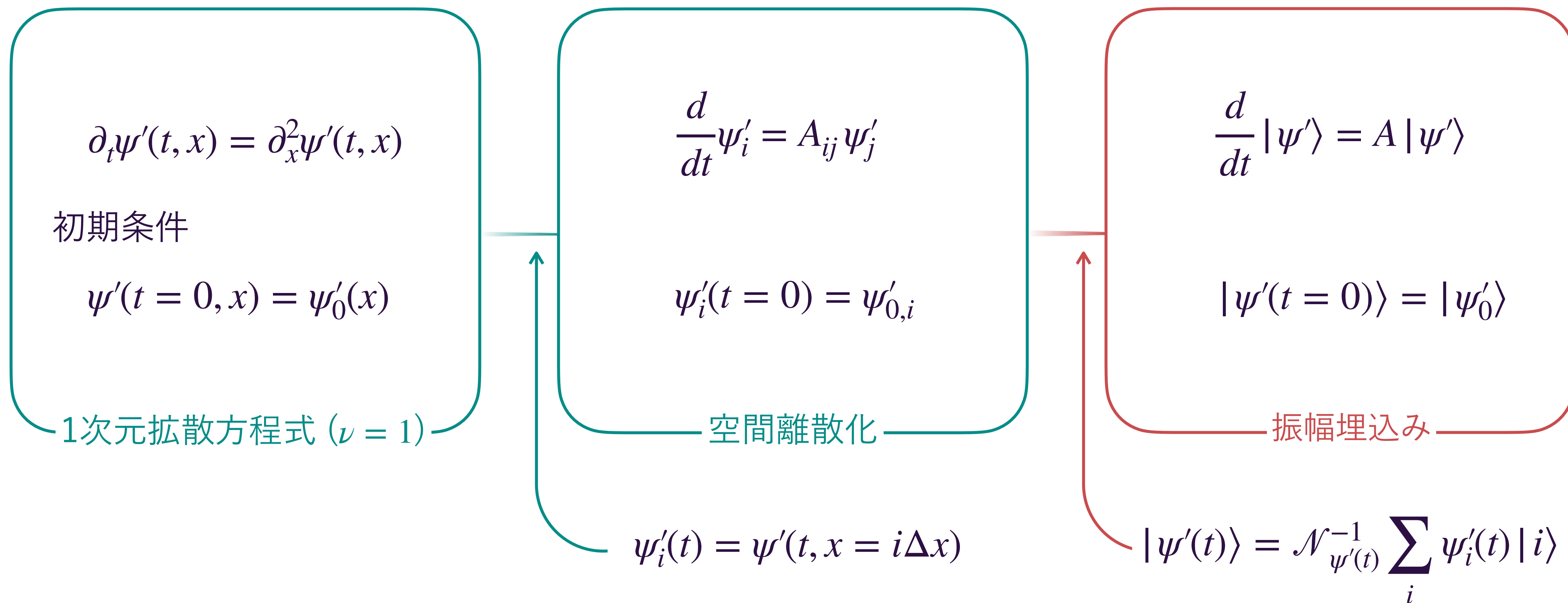


- 提案アルゴリズムの概要
- 2点関数の効率的な読み出し
- 一般化 (多点関数・近似の改良)

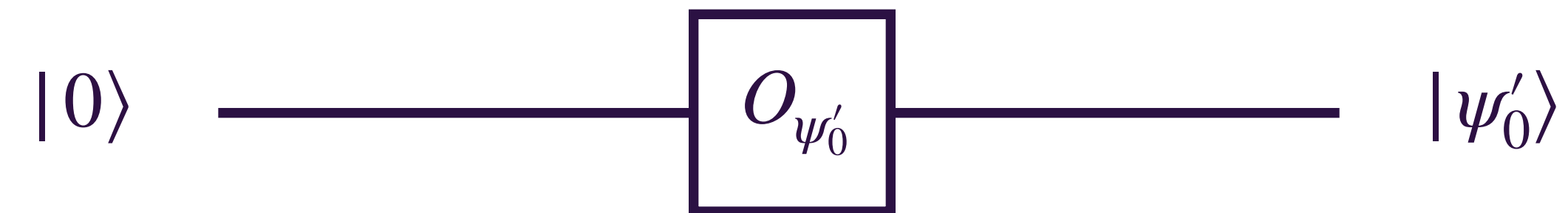
時間積分

Burgers 方程式のかわりに拡散方程式を積分する

情報読出し時の都合のために、 $\psi' := \partial_x \psi$ について解くことにする



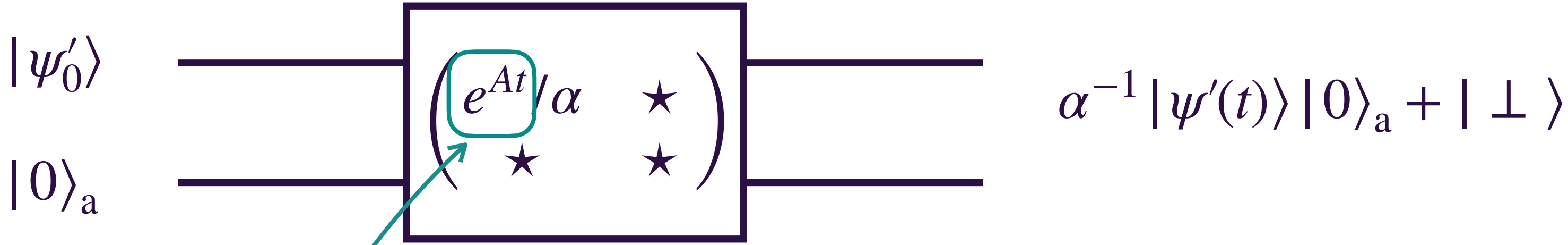
初期状態生成は、オラクルを仮定



初期条件 $|\psi'_0\rangle$ を効率的に生成する oracle $O_{\psi'_0}$ の存在を**仮定**する。

計算量評価では $O_{\psi'_0}$ の呼び出し回数を数える。

時間発展は、block encoding 中心差分スキーム × 周期境界条件



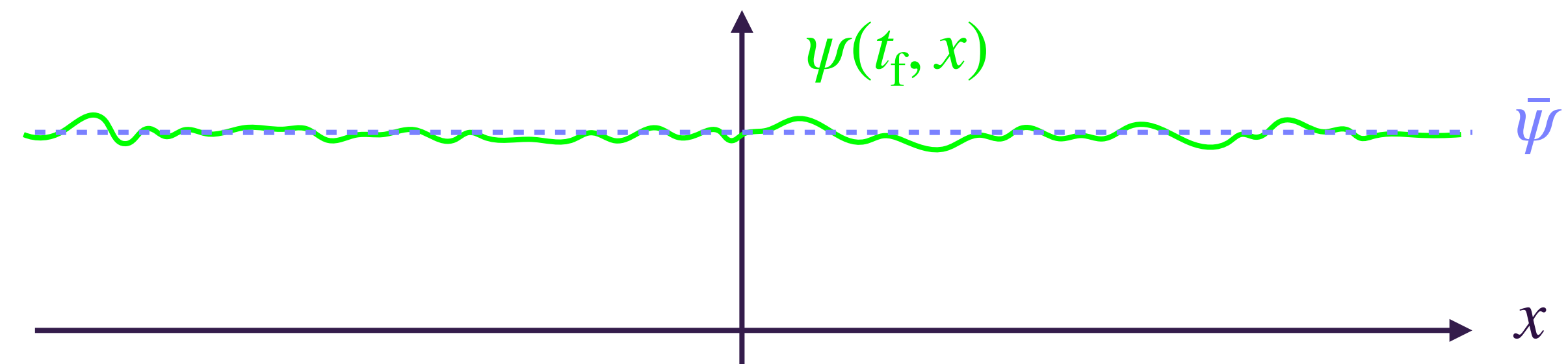
block encoding ※ とくに linear combination of Hamiltonian simulation [An, Childs, Lin 2026]

$$\psi'_i(t) = \underbrace{(e^{At})_{ij}}_{\text{非ユニタリ}} \psi'_{0,j}, \quad A_{ij} = \begin{pmatrix} -2 & 1 & 0 & \dots & 1 \\ 1 & -2 & 1 & \dots & 0 \\ 0 & 1 & -2 & \dots & 0 \\ & & \dots & & \\ 1 & 0 & 0 & \dots & -2 \end{pmatrix}$$

統計量の読出し

逆 Cole–Hopf 変換を線形近似

最終時刻で $\psi(x)$ が摂動的であることを仮定すると



$$u(t_f, x) = -2 \frac{\partial_x \psi(t_f, x)}{\psi(t_f, x)} \simeq -2 \frac{\partial_x \psi(t_f, x)}{\bar{\psi}} \propto \partial_x \psi(t_f, x)$$

この近似のもとで

$$|u(t_f, x)\rangle = |\partial_x \psi(t_f, x)\rangle$$

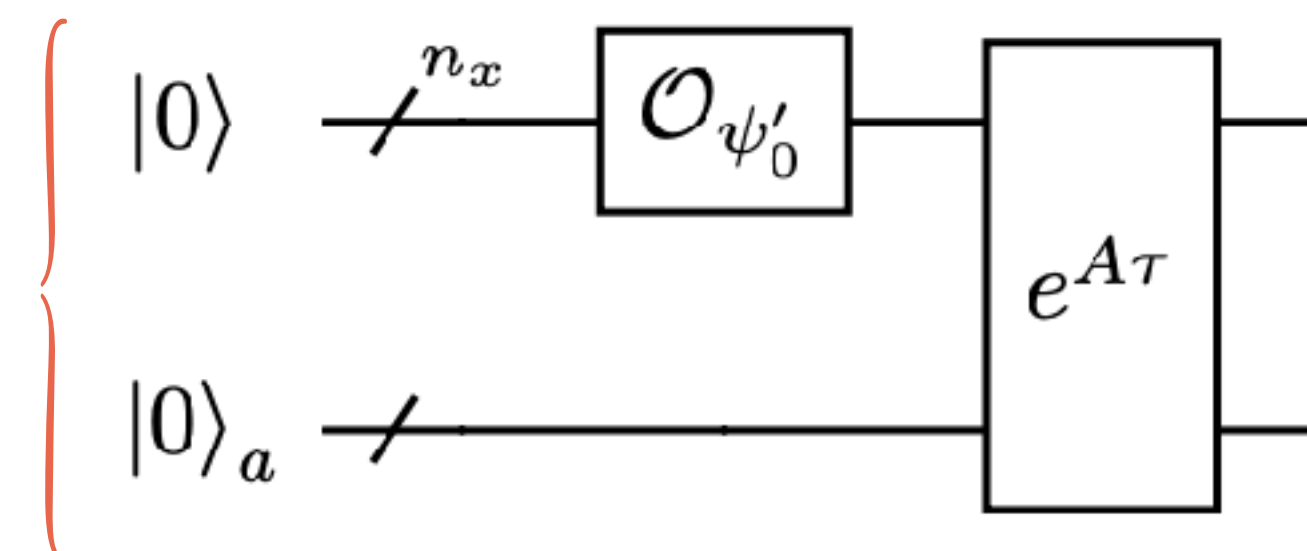
2点関数

$$\begin{aligned}
 P^{(2)}(r) &:= \overline{u(x) u(x+r)} = N_x^{-1} \sum_i u_i u_{i+\rho} \\
 &= \frac{N_x^{-1} \|u\|^2 \langle u | C^{(2)}(\rho) | u \rangle}{= \mathcal{O}(1)}
 \end{aligned}$$

許容誤差 ε とすると

$$\frac{P^{(2)}(r)}{P^{(2)}(0)} = \frac{\langle u | C^{(2)}(\rho) | u \rangle}{\langle u | C^{(2)}(0) | u \rangle}$$

計算量 $\sim 1/\varepsilon$



$$\begin{aligned}
 C^{(2)}(\rho) &= \frac{1}{2} (P_{N_x}^\rho + P_{N_x}^{-\rho}), \\
 P_{N_x} &:= \sum_{j=0}^{N_x-2} |j+1\rangle \langle j| + |0\rangle \langle N_x-1|
 \end{aligned}$$

overlap estimation algorithm で測定

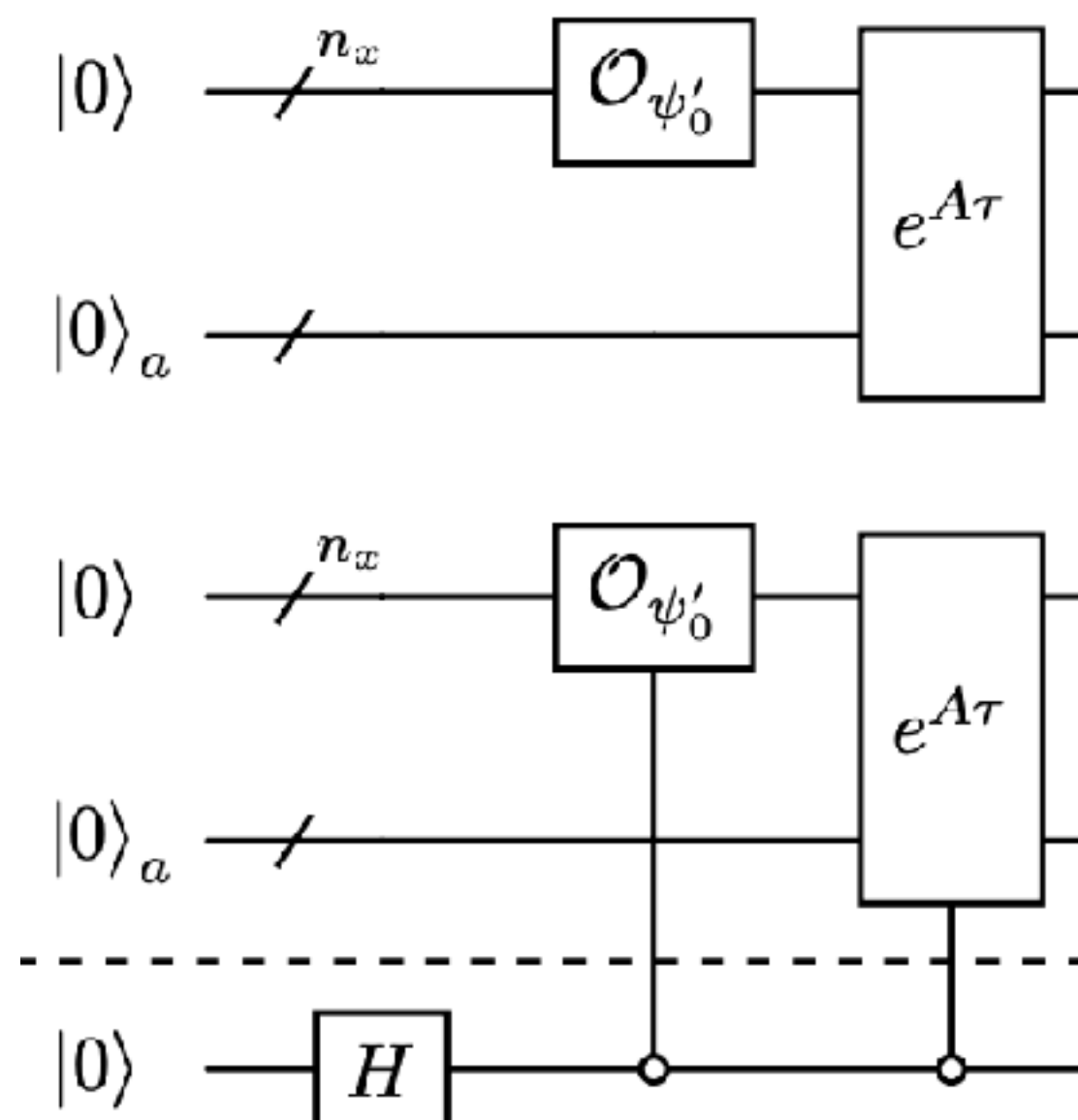
多点関数へ 3点関数の場合

$$\begin{aligned}
 P^{(3)}(r_1, r_2) &:= \overline{u(x) u(x+r_1) u(x+r_2)} = N_x^{-1} \sum_i u_i u_{i+\rho_1} u_{i+\rho_2} \\
 &= \frac{N_x^{-1} \|u\|^3 \langle U^{(3)} | C^{(3)}(\rho_1, \rho_2) | U^{(3)} \rangle}{\mathcal{O}(N_x^{1/2})}
 \end{aligned}$$

許容誤差 $N_x^{-1/2} \epsilon$

$$\frac{P^{(3)}(r_1, r_2)}{P^{(3)}(0, 0)} = \frac{\langle U^{(3)} | C^{(3)}(\rho_1, \rho_2) | U^{(3)} \rangle}{\langle U^{(3)} | C^{(3)}(0, 0) | U^{(3)} \rangle}$$

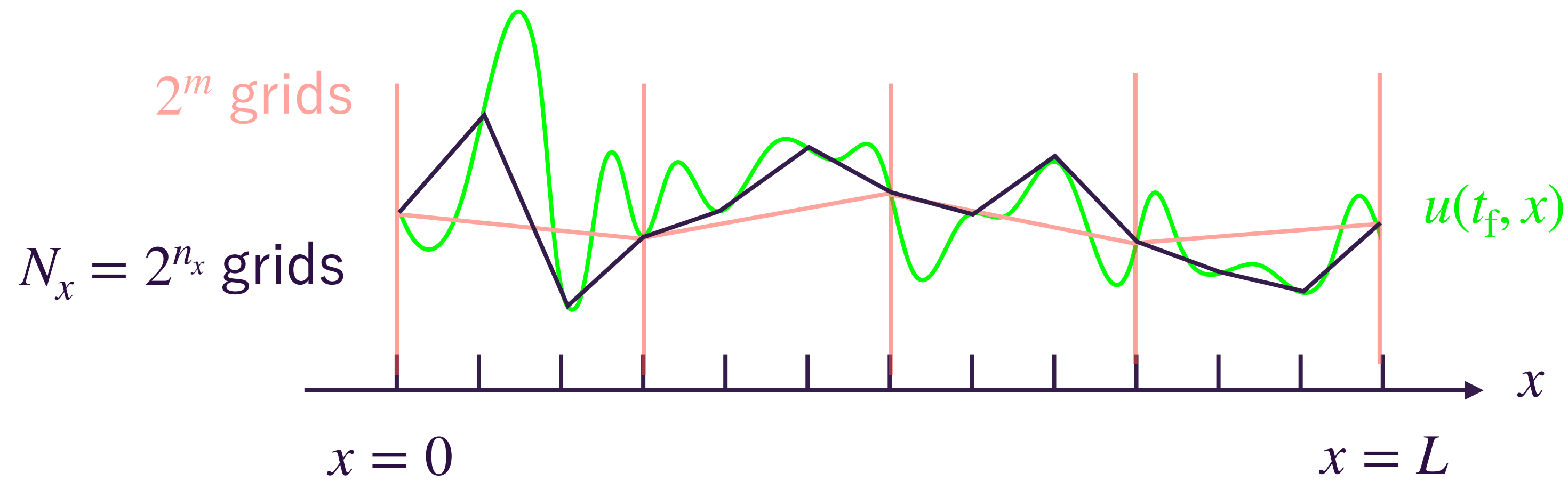
計算量 $\sim N_x^{1/2} / \epsilon$



$$\begin{aligned}
 |U^{(3)}\rangle &:= \frac{|\mathbf{u}\rangle^{\otimes 2} \otimes |0\rangle + |\mathbf{u}\rangle \otimes |\mathbf{0}\rangle \otimes |1\rangle}{\sqrt{2}} \\
 &= \sum_{j,k} \frac{u_j u_k}{\sqrt{2}} |j, k, 0\rangle + \sum_l \frac{u_l}{\sqrt{2}} |l, 0, 1\rangle + |\perp\rangle
 \end{aligned}$$

overlap estimation algorithm で測定

多点関数を調べる場合には適宜粗視化

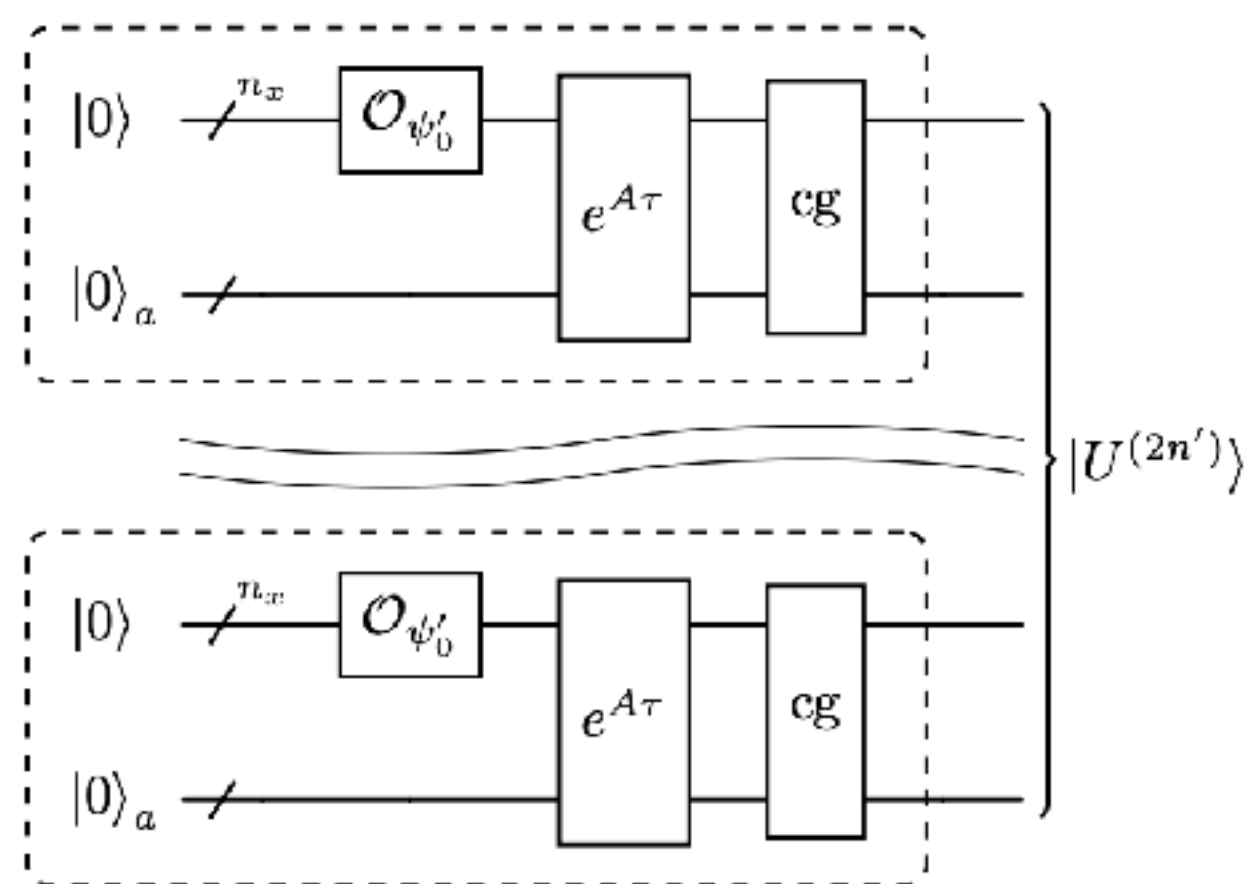


$$\begin{aligned}
 |\mathbf{u}\rangle_{\text{cg}} &:= 1_{2^m} \otimes H^{\otimes n_x - m} |\mathbf{u}\rangle \\
 &= \sum_{k=0}^{2^m - 1} u_{\text{cg } k} |k\rangle |0\rangle_{\text{ss}} + |\perp\rangle,
 \end{aligned}$$

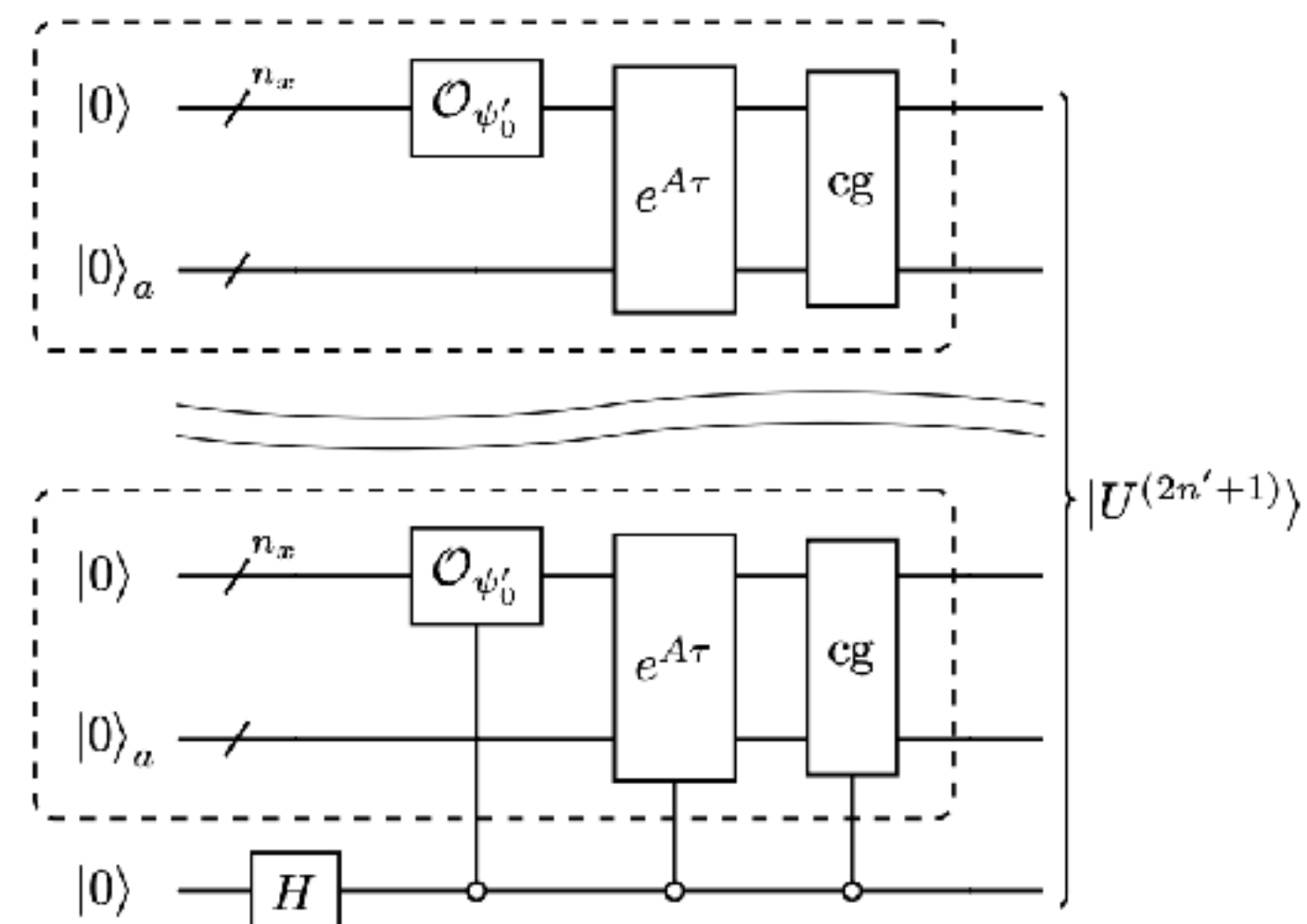
$$u_{\text{cg } k} := 2^{-\frac{n_x - m}{2}} \sum_{j=2^{n_x - m} k}^{2^{n_x - m} (k+1) - 1} \frac{u_j}{\|\mathbf{u}\|}$$

多点関数へ

$n = 2n' : \text{even}$



$n = 2n' + 1 : \text{odd}$



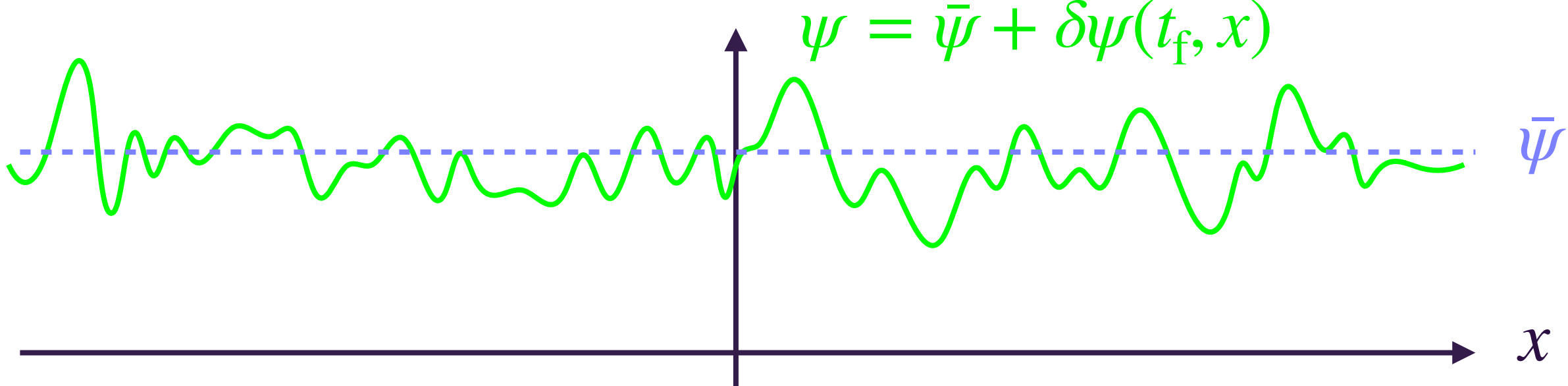
$$P^{(n)}(r_1, \dots, r_n) = \underbrace{2^{-m} 2^{-n(n_x - m)/2}}_{= \mathcal{O}(2^{(n-2)m/2})} \underbrace{\|u\|^n \langle U^{(n)} | \tilde{C}^{(n)}(\rho_1, \dots, \rho_n) | U^{(n)} \rangle}_{\text{許容誤差 } 2^{-(n-2)m/2} \varepsilon \text{ で測定}}$$

$$\frac{P^{(n)}(r_1, \dots, r_2)}{P^{(n)}(0, \dots, 0)} = \frac{\langle U^{(n)} | \tilde{C}^{(n)}(\rho_1, \dots, \rho_n) | U^{(n)} \rangle}{\langle U^{(n)} | \tilde{C}^{(n)}(0, \dots, 0) | U^{(n)} \rangle} \quad \text{計算量} \sim 2^{(n-2)m/2} / \varepsilon$$

近似の改良

高次の近似

読み出し時に $\frac{\delta\psi}{\bar{\psi}} \sim \frac{(\mathbf{u} \cdot \nabla) \mathbf{u}}{\nu \nabla^2 \mathbf{u}} = \text{Re} \lesssim 1$ を仮定



0 次近似

$$u = -2\nu \frac{\partial_x \psi}{\psi} \simeq -2\nu \frac{\partial_x \psi}{\bar{\psi}} \quad \blacktriangleright \quad |u(t_f, x)\rangle = |\partial_x \psi(t_f, x)\rangle$$

1 次近似

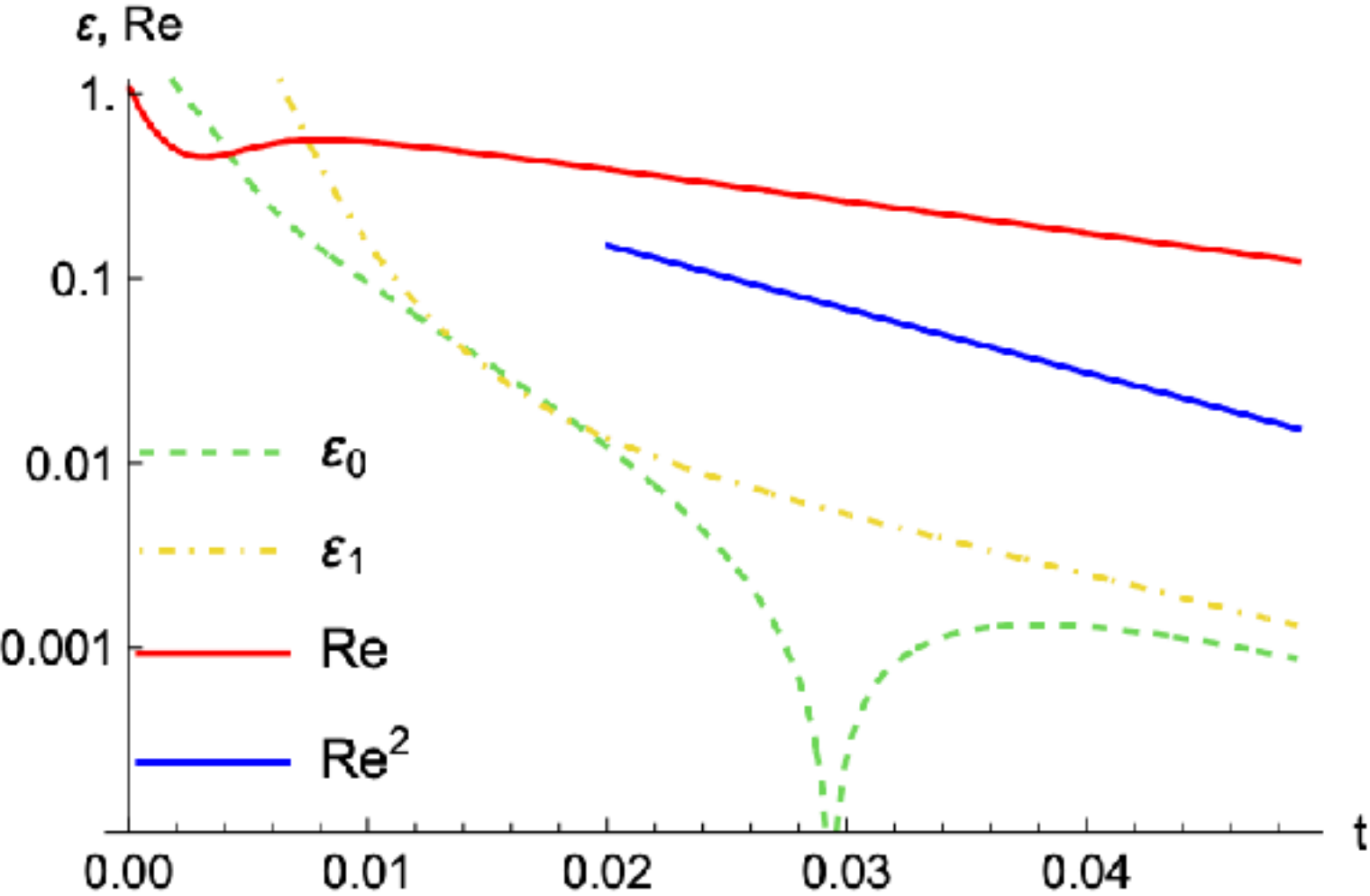
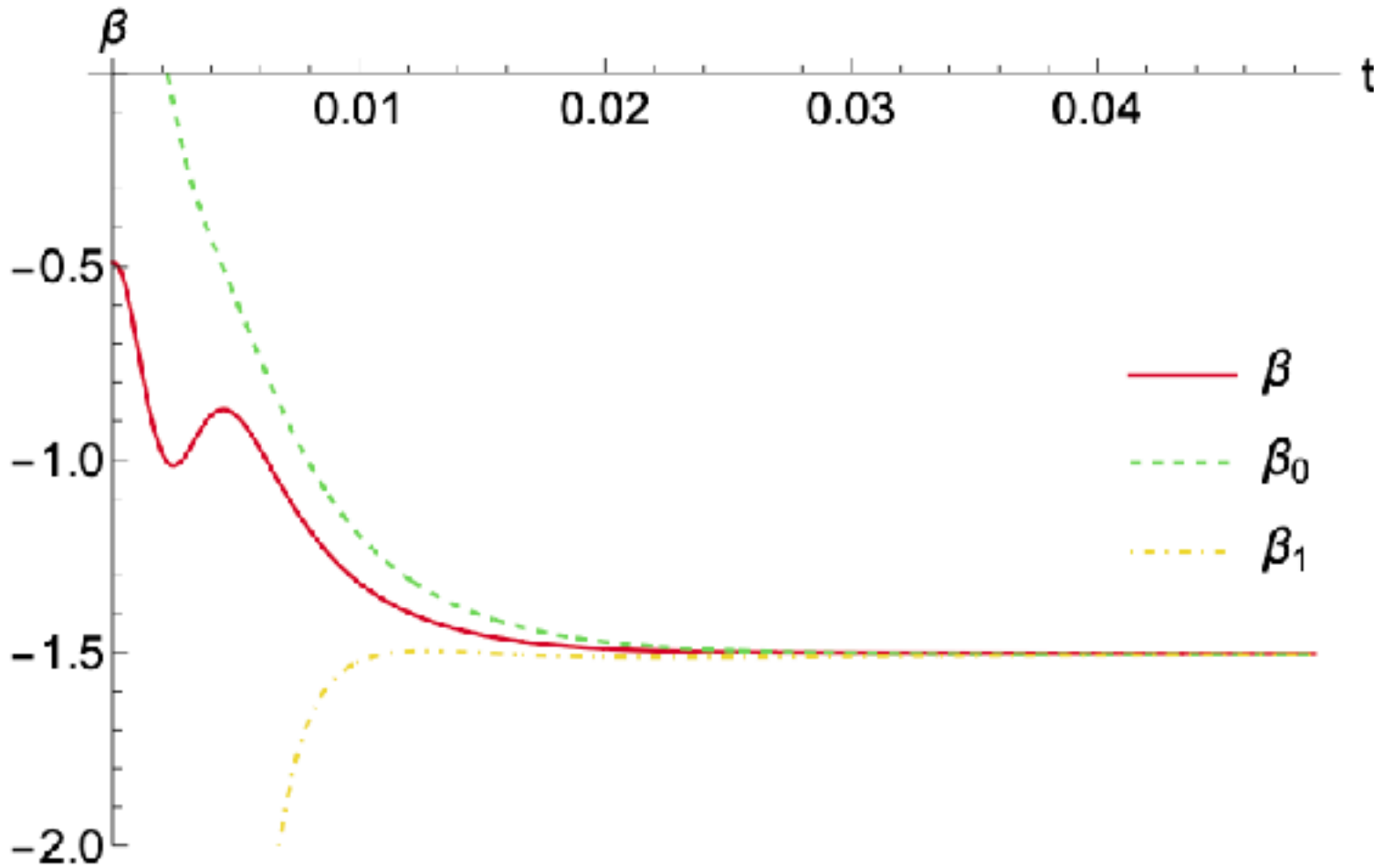
$$u = -2\nu \frac{\partial_x \psi}{\psi} \simeq -2\nu \frac{\partial_x \psi}{\bar{\psi}} + 2\nu \frac{\partial_x \psi \delta\psi}{\bar{\psi}^2} \quad \blacktriangleright \quad |u(t_f, x)\rangle = |\phi(t_f, x)\rangle |\partial_x \psi(t_f, x)\rangle$$

$\phi := \bar{\psi} - \delta\psi$

Flatness のエラー

$$\beta := \frac{P^{(4)}(0)}{P^{(2)}(0)^2} - 3 \quad \text{flatness}$$

$$\varepsilon := \left| \beta_{\text{approx}} / \beta - 1 \right| \quad \text{relative error}$$



知りたい統計量によっては 0 次近似で十分

計算量評価

古典計算との比較

2点関数の評価は N_x について効率的!

$$u = -2\nu \frac{\partial_x \psi}{\psi}, \quad \psi(t, x) = \frac{1}{\sqrt{4\pi t}} \int dy \exp\left(-\frac{(x-y)^2}{4t}\right) \psi_0(y)$$

古典的に評価するためには、ナイーブには $\mathcal{O}(N_x)$,

n 点関数 $P^{(n)} = L^{-1} \int dx u(x)u(x+r_1)\cdots u(x+r_{n-1})$ を評価するには $\mathcal{O}(N_x^2)$.

一方、今回の量子アルゴリズムでは、
 $\sim t_f \frac{\|\psi'_0\|}{\|\psi'(t_f)\|} \frac{\log N_x \cdot 2^{(n-2)m/2}}{\varepsilon}$

積分時間 (green dot) n 点関数 (green dot) 粗視化しない場合 $m = \log N_x$ (red dot)
空間体積 (red dot) 許容誤差 (green dot)

まとめ

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- 宇宙論では、ゆらぎの統計量が意味を持つ
- 統計量 (多点関数) のみ読出せれば十分 → 量子計算の困難克服の可能性

- 今回は、1次元バーガス方程式に対する量子計算アルゴリズムを提案
 - 非線形変数変換により、拡散方程式に帰着 (厳密に線形化)
 1. 初期条件は、オラクルを仮定
 2. 時間発展は、linear combination of Hamiltonian simulation
 3. オペレータの期待値として統計量を読出し (overlap estimation)
 - 2点関数の評価は N_x について指数加速

 - ただし重要な仮定：初期状態準備オラクルの存在, 読出し時に $\text{Re} \ll 1$ の近似